Multiple job holding: the artist's labor supply approach

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1) Introduction

This paper presents a generalization of the static labor supply model, in which an individual may hold two jobs, one of which brings utility in itself; we term it "artistic job". Additionally, we allow for earnings in the artistic job to be non linear in hours dedicated to it. Two strands of literature relate to this problem, the "moonlighting" or dual job holding, and the artists' labor supply literature.

One approach in the moonlighting literature views the decision to hold a secondary job as resulting from a constraint on hours worked in the primary job (Shishko and Rostker, 1976). Smith Conway and Kimmel (1997) present a utility maximization model in which workers perceive both jobs as heterogeneous; hence hours worked on both jobs enter the utility function.

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In the cultural economics literature the current standard reference on how artists supply labor to arts and non-arts markets is Throsby's (1994) work-preference model. [2] This author presents the problem of an agent that excerpts utility from consumption and artistic work and has a time constraint and an income constraint. In its "strong" version, assumptions imply that artists' are overwhelmingly motivated to create art, only care about a minimum consumption bundle and do not care at all about leisure. Artists would only choose non-arts work to complement artistic income in order to attain the minimum consumption level. The resulting work-preference results not from utility maximization but from an operation on the budget constraint. In a weaker version (also discussed in Caserta and Cuccia (2001)), the minimum consumption condition is not imposed, but leisure is not considered in the utility function; hence changes in arts work also translate unambiguously into non arts labor supply.

On empirical grounds, Filer (1986) suggests that the starving artist is more a myth than a real life character. Artists as everybody else care about consumption and therefore it is possible that an increase in non-arts market wage may induce artists to supply less arts time and to consume more. Throsby's (1994) model predicts that an increase in the non-arts wage will induce an increase in arts labor supply, but his model's framework does not accommodate the whole set of possible artist's time allocations. One of the main departures of our model with respect to this literature is that (besides arts work time) we allow for leisure in the utility function and time constraint of agents.

Artists may love their work. However, though dedicated to creation, they may also distinguish between life and art. Hence, somewhat paradoxically, we rely on a conventional model, closer to home production and labor supply models, to provide valuable insights on the behavior of this unconventional occupational group. The implications are considerable, since arts policy relies massively on incentives to personal dedication, and its impacts are carried out decisively through entry and exit decisions. The precise behavioral response assessment and the possible existence of "marginal artists" are key elements of policy design. Our results are also interesting to other activities or occupations in which the "doing what you love" ingredient is important, such as the academic field or volunteer work.[3]

This paper contributes to the literature in several ways. First, it solves the problem of a utility maximizing agent that cares about consumption, leisure and artist work. Second, it allows artistic income to be a non linear function of arts labor and of market perceived artistic quality. Third, it tests the empirical predictions of the model.

2) The model

Unlike the traditional labor supply models, in our model the individuals derive pleasure from arts time and therefore it is an argument in the utility function. This is a feature of Throsby's (1994) model. Unlike Throsby's, our model includes leisure as a separate

[2] See Menger (1999). An early unpublished reference is Hamermesh (1974) who studies the behavior of the traditional labor supply model in the case in which individuals enjoy their work.

[3] See, for instance, García and Marcuello (2002) for a model for contributions to volunteer work.

argument. In the specific case of artists, it may be argued that the limit between leisure and work is tenuous. However, American novelist Philip Roth states that art must draw on life, and complete dedication to art led him to miss life.[4] We keep this distinction trying to capture the presence of all other time allocation uses. [5] Leisure is not identified with arts time.

The individuals have two sources of income: the non arts and the arts activities. Artistic earnings may not be linear in the hours supplied to arts work. Throsby (1994) comments that this can be the case in artists that produce works that are to be sold as such. In the case of performing artists, they may face downward sloping demand curves for the number of performances supplied; hence more output might be linked to decreasing marginal income. [6]Moreover, artistic ability may be seen as a fixed factor and arts time as subject to diminishing returns.[7] Therefore in our model we allow arts income to be a non linear function of arts hours and of market perceived artist quality. This has been suggested before but we know of no paper in which it has been explored neither theoretically nor empirically. This also separates our model from that of Smith and Kimmel (1998), in which hours of both jobs enter the utility function and wage schedules are linear in both cases.

In our model an individual will maximize a concave utility function $U(c,l,h_A)$ where c represents an aggregate consumption basket, l stands for leisure and h_A is time devoted to art. The time constraint implies that total time (T) has to be divided between leisure (l), arts time (h_A) and non arts work (h_N). In the budget constraint, without loss of generality, we normalize the price of the consumption bundle to 1. Income is derived from three sources: working in the non arts labor market at a wage rate w, arts work, and non-hours related sources of income. Arts income is assumed to depend positively on arts hours and a measure of market perceived quality. We understand by market perceived quality all those features of artistic output that can potentially shift the arts earnings, and not the artists' perceived quality or critic's aesthetic valuation. We specifically allow for non-linearities in this source of income.

Income not related to working hours is typically associated with rents from assets or transfers. An artist may also receive royalties for her past creations not necessarily related to current artistic effort. Since they would enter equally in the utility function, we consider the combined effect of the sum of all non labor income, V.

Formally, the maximization problem would be:

[4] "It was the interests in life and the attempt to get life down on the pages which made me a writer - and then I discovered that, in many ways, I am standing on the outside of life". Philip Roth, interview in The Guardian, London, Dec 14th, 2005.

[5] Uruguayan writer Mario Levrero documents in a novel the subtle psychological changes of a writer that receives a grant that will let him finally write freely his work, and the conflicts surrounding time allocation and transitions between writing and non writing time. Mario Levrero, La novela luminosa *(The Enlightening Novel)*, Alfaguara, 2005.

[6] See for instante a model of self employed physicians as owner operated firms in Thornton (1998).

[7] In the long run, arts activity has a human capital dimension, dedication enhances ability, etc. We only tackle the static problem.

$$\underset{c,l,h_A}{Max} \quad U(c,l,h_A)$$

subject to: $\quad w(T-l-h_A)+f(h_A,\theta)+V=c\,; \qquad l+h_A \le T\,; \quad l \ge 0\,; \quad h_A \ge 0\,; \quad c \ge 0\,.$

Assumptions (1):

$f_1 > 0, \quad f_{11} < 0, \quad f_2 > 0, \quad f_{12} > 0\,; \qquad U_c(\cdot) > 0, \quad U_l(\cdot) > 0, \quad U_{h_A}(\cdot) > 0 \quad ;$

$U_{cc}(\cdot) < 0, \quad U_{ll}(\cdot) < 0, \quad U_{h_A h_A}(\cdot) < 0$ or $\partial U(X)/\partial^2 X$ negative semidefinite, with $X = (c,l,h_A)$.

Assumption (1) implies that arts income is increasing in arts hours at a decreasing rate (concavity), and it is increasing in the market perceived quality. Hours and market quality are complements (artists more attractive to the market have larger marginal income per hour). We also assume that Inada conditions hold, ruling out corner solutions, for consumption, leisure and arts time. Formally:

$$\lim_{c\to 0} U_c(\cdot) = \infty; \quad \lim_{l\to 0} U_l(\cdot) = \infty; \quad \lim_{h_A\to 0} U_{h_A}(\cdot) = \infty.$$

The first two are standard in utility maximization models, and the third is a natural extension of this criterion. [8]

Given this conditions, the problem can be stated as maximizing the following Lagrangian:

$$\ell = U(c,l,h_A) + \mu_0\big(c - w(T-l-h_A) - f(h_A,\theta) - V\big) + \mu_1(T-l-h_A) + \mu_2 l + \mu_3 h_A + \mu_4 c$$

where the first order Kuhn Tucker conditions are as follows:

$\partial\ell/\partial c = 0\,; \qquad \partial\ell/\partial l = 0\,; \qquad \partial\ell/\partial h_A = 0\,;$

$\partial\ell/\partial\mu_0 = 0\,; \qquad \partial\ell/\partial\mu_1 \ge 0\,; \qquad \partial\ell/\partial\mu_2 \ge 0\,; \qquad \partial\ell/\partial\mu_3 \ge 0\,; \qquad \partial\ell/\partial\mu_4 \ge 0\,;$

$\mu_0 \ge 0\,; \qquad \mu_1 \ge 0\,; \qquad \mu_2 \ge 0\,; \qquad \mu_3 \ge 0\,; \qquad \mu_4 \ge 0$

$\mu_0[c - w(T-l-h_A) - f(h_A,\theta) - V] = 0\,; \qquad \mu_1(T-l-h_A) = 0\,;$

$\mu_2 l = 0\,; \qquad \mu_3 h_A = 0\,; \qquad \mu_4 c = 0\,.$

Inada conditions imply that at the optimum $l > 0$; $c > 0$ and $h_A > 0$, so μ_2, μ_3 and μ_4 must

[8] If this condition does not hold the model besides the full time and part time artist, could potentially nest a third case, the non artist, corresponding to the standard static labor supply model.

equal 0.

Like all static maximization problems where agents enjoy consumption, they spend their entire budget. For any arts and non arts hours choice, implying certain income, agents do not derive utility by saving. Thus, the budget constraint will be satisfied with equality, so we do not worry about $\mu_0 \geq 0$. Our problem simplifies to:

$$U_C + \mu_0 = 0 \qquad \textbf{(1)};$$

$$(T - l - h_A) \geq 0; \qquad \mu_1 \geq 0; \qquad \mu_1 (T - l - h_A) \geq 0;$$

$$U_l + \mu_0 w - \mu_1 = 0 \qquad \textbf{(2)};$$

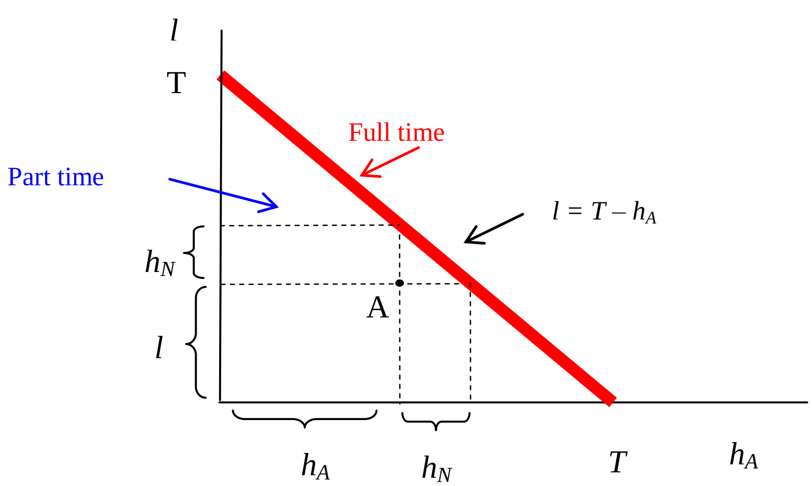
$$U_{h_A} + \mu_0 (w - f_1) - \mu_1 = 0 \qquad \textbf{(3)};$$

$$c - w(T - l - h_A) - f(h_A, \theta) - V = 0 \qquad \textbf{(4)}.$$

3) Graphic representation

A simple two dimensional representation of the time constraint is a triangle with sides of length T and its vertex at origin in the plane (l, h_A) as in Figure 1.

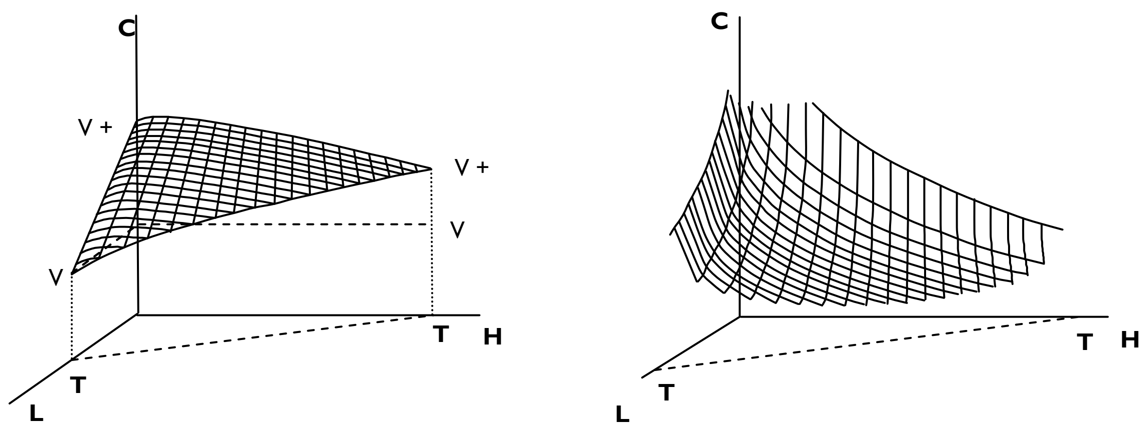
Figure 1. Time allocation set



All points on the straight line $l = T - h_A$ correspond to full time artists and imply $h_N = 0$. An individual in a point like A defines l and h_A, and then h_N is uniquely determined by the distance to the constraint.

In a three dimensional graph in space *(c, l, h$_A$), t*o each point in the triangle of feasible time allocations corresponds a point in the budget surface $c = w(T - l - h_A) + f(h_A, \theta) + V$. Given h_A, increases in *l* translate in reductions in non arts work time at rate −*w*, the slope of a slice of the budget surface parallel to the *l* axis. Given *l*, increases in h_A correspond to reductions in non arts work time valued at *w*, but arts income is generated at a rate $f(h_A, \theta)$, so the slope of a slice of the budget surface parallel to the h_A axis is equal to $f_1 - w$. It can be the case that $f_1 > w$ for all values of h_A, in which case the slope will always be positive, and will always be negative if conversely $w > f_1$. Finally, a third case is when this slope is initially positive and becomes negative after a certain threshold. Constant utility levels define indifference surfaces in the space *(c, l, h$_A$)*. Such surfaces will be convex towards the origin (see Figure 2). An interior solution implies the tangency between the budget surface and an indifference surface in the space *(c, l, h$_A$)*.

Figure 2. Budget constraint and indifference surface



Choices can also be represented through subsets of the space *(c, l, h$_A$)* in which one of the variables is kept constant. Graphically, this would yield “slices” of the original three-dimensional graph parallel to the axes.

4) The full time and part time artists cases

Two cases are of interest to this paper, the full time and the part time artist, characterized as follows:

1. Part time artist

For a part time artist: $h_N > 0, h_A > 0$. Then $T - L - h_A > 0 \quad \mu_1 = 0$. From (1) and (2) we get
$U_l / U_c = w$ **(5).**
From (1) and (3) we obtain $U_{h_A} - (w - f_1(\cdot))U_c = 0 \quad \Rightarrow \quad U_{h_A} / U_c = w - f_1(\cdot)$ **(6).**

Equation (6) states that for a part time artist, marginal utility of arts hours should equal the loss in marginal consumption due to switching from non arts to arts work. Euler conditions equating the marginal rates of substitution and marginal rates of transformation are obtained for consumption and leisure (equation (5)) and consumption and arts time (equation (6)). Combining (5) and (6):

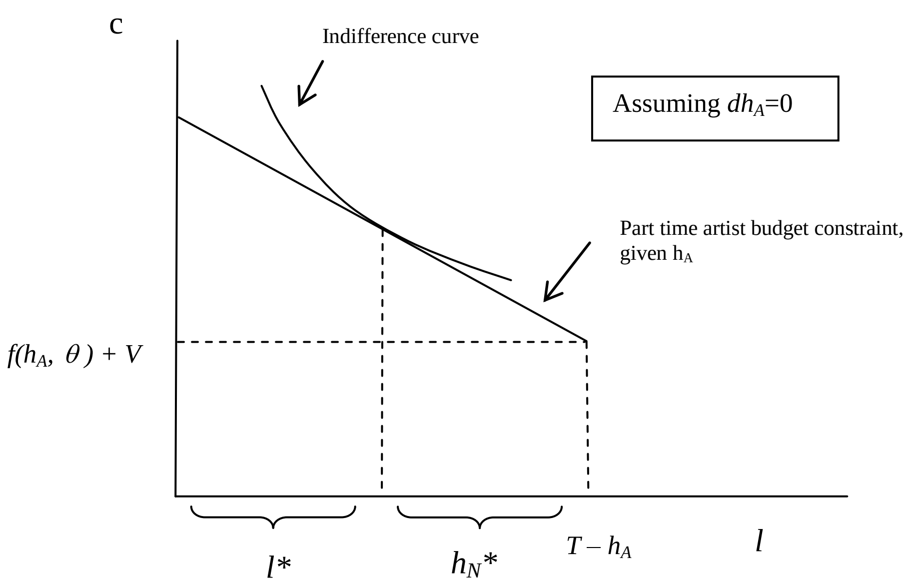
$$U_l = U_{h_A} + U_c f_1(\cdot) \qquad \textbf{(7)}$$

Utility of one more hour dedicated to leisure should equal the direct utility of one more hour dedicated to art plus the marginal utility of consumption derived from the increase of arts income. Finally we restate the budget constraint

$$c = w(T - l - h_A) + f(h_A, \theta) + V \qquad \textbf{(8)}$$

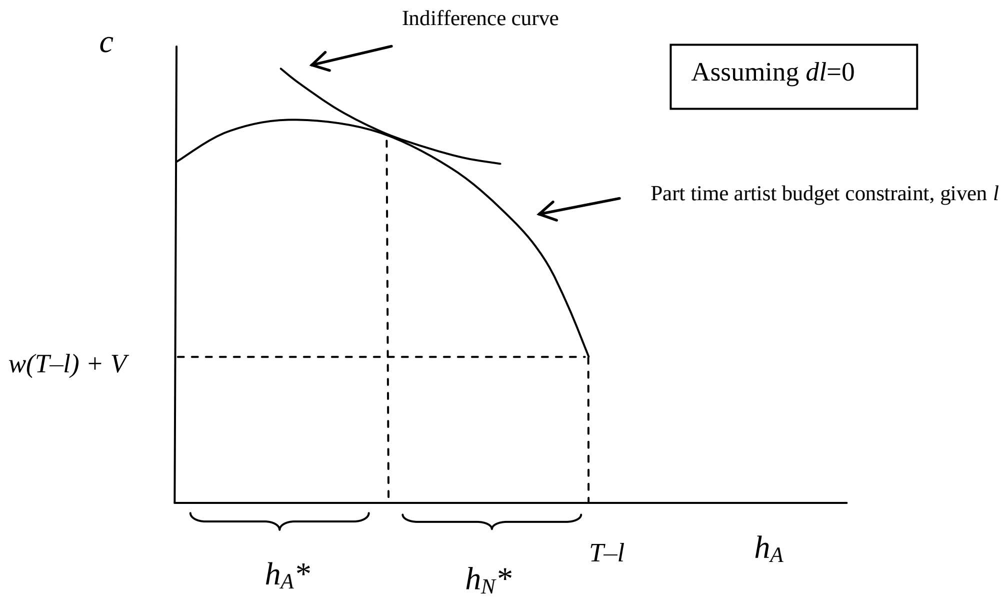
Equations (7), (8) and (9) determine c, l, h_A. The case in which the individual would choose an interior point is displayed in figure 3 as a slice of the plane (*l*, *c*).

Figure 3. Part time artist, *c-l* trade off given h_A



The slope of the indifference curve is $-U_l / U_c$ and the slope of the budget constraint is $-w$. The tangency point corresponds to equation (7) for part time artists determining leisure and by difference non arts hours (see Figure 4).

Figure 4. Part time artist, *c*-*h*$_A$ trade off given *l*



The slope of the budget corresponds to the difference $f_1(\cdot)-w$, which in this case was drawn to increase initially and then to decrease after a certain point. The slope of the indifference curve pictured is $-U_{hA}/U_c$. The tangency point corresponds to equation (6) for part time artists determining arts hours and by difference non arts hours.

2. Full time artist

By definition of full time artist $h_N=0, h_A>0$ and $h_A=T-L$. As in the previous two cases we can use (1) and (2) to eliminate the Lagrange multiplier obtaining:

$$U_l - U_c w = \mu_1 > 0 \quad \Rightarrow \quad U_l/U_c > w \qquad \textbf{(9)}$$

From (1) and (3) we get $U_{h_A} - U_c w + f_1(\cdot)U_c = \mu_1 > 0$ **(10)**

and from (10) and (11) we find as in equation (8): $U_l = U_{h_A} + U_c f_1(\cdot)$ **(11).**

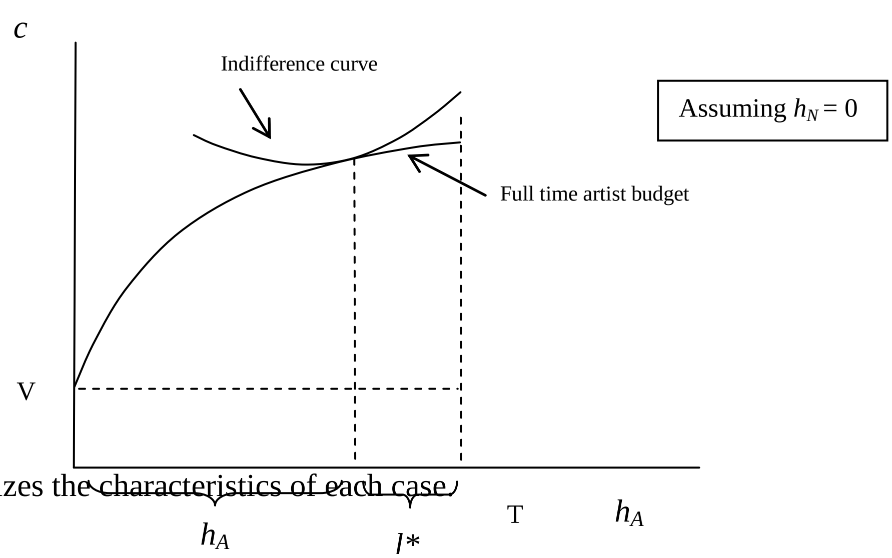
The budget constraint simplifies to: $c = f(h_A, \theta) + V$ **(12).**

Budget constraint (12), equation (11) and condition $T = l + h_A$ uniquely determine c, l, h_A . For full time artists, we have a special indifference map. Linked by the time constraint, leisure and arts hours are no longer independent. A full time artist is located exactly over the segment TT in figure 1, i.e. $T = l + h_A$. This determines in the (c,h_A) plane, a strictly increasing budget set, with slope f_1. Indifference curves can be alternatively defined between the pairs of goods (c,l) or (c,h_A), since the condition $T = l + h_A$ implies that choice of l uniquely determines h_A and vice versa. Our three variable space collapses to a two-dimensional one, and $U(c,l,h_A)$ becomes $U(c,h_A)$.

For the full time artist, at the optimum $U_c dc + U_l dl + U_{h_A} dh_A = 0$. As $-dl = dh_A$, then it must hold $U_c d\,c + (U_{h_A} - U_l)dh_A = 0$. Then the slope of an indifference curve in the plane *(c, h_A)*, is given by $d\,c/dh_A \quad = (U_l - U_{h_A})/U_c$ .In this case indifference curves are not monotonic. For small h_A, marginal utility of arts work net of leisure is positive, hence $U_{h_A} - U_l > 0$ and the indifference curve in the *(h_A, c)* space has a negative slope. However, for large values of h_A, l would be relatively small and the opposite will be true: $U_l - U_{h_A} > 0$. As from equation (10) we know that $(U_l - U_{h_A})/U_c = f_1 > 0$ i.e. the optimum must lie in the region where the slope of the indifference curve is positive.

Indeed, this is the region in which there is a true trade off for the artist. To the left of the minimum of the indifference curve the artist gains by reducing leisure, since obtains larger consumption but also the time trade off is advantageous. At the minimum of the indifference curve, $dc/dh_A = 0 \Leftrightarrow U_{h_A} = U_l$. Given concavity, to the left of this point $U_{h_A} > U_l$, i.e. for each hour subtracted to leisure to be dedicated to arts work, utility gain is larger than the corresponding loss. When arts time is long enough the trade off appears.

Figure 5. Full time artist *c-h_A* trade off



<u>*3. Summary*</u>

Table 1 summarizes the characteristics of each case

Table 1 **Characteristics of optima, part time and full time artists**			
Part time artists		Full time artists	
$U_l / U_c = w$	(5)	$U_l / U_c > w$	(9)
$U_{h_A} / U_c = w - f_1(\cdot)$	(6)	$U_{h_A} / U_c \geq w - f_1(\cdot)$	(10)
$(U_l - U_{h_A}) / U_c = f_1(\cdot)$	(7)	$(U_l - U_{h_A}) / U_c = f_1(\cdot)$	(11)
$c - w(T - l - h_A) - f(h_A, \theta) - V = 0$	(8)	$c - f(h_A, \theta) - V = 0$	(12)

Both types of agents have in common the equalization between the difference of marginal rates of substitution (between leisure and consumption and arts time and consumption) and marginal arts income. However, in the case of full time artists, they do not consider non arts market because: 1. marginal rate of substitution between leisure and consumption is larger than the foregone non arts wage, and 2. marginal rate of substitution between arts time and consumption is larger than the foregone difference between non arts wage and marginal arts income.

5) Comparative static

Some statements that do not require more restrictive assumptions are the following (proofs are given in the appendix):

Proposition 1
Under A1,
a. Marginal changes in w do not modify the behavior of full time artists.
b. Part time artists could be affected both by changes in w and changes in θ.

This can be stated in terms of the traditional labor supply framework: changes in factor returns induce supply increases (substitution effect between consumption and leisure) or supply reductions (income effect, consumption and leisure). This applies both to “leisure consumption” and to “arts time consumption” (as long as h_A is within the relevant range).

Reservation wage
Part a. of proposition 1 refers only to marginal changes in wages. Should wages increase enough, it is possible that a full time artist becomes a part time one.

Proposition 2
Under A1, there is a reservation wage w_R *that -ceteris paribus- makes an individual indifferent between being a full time or part time artist. The reservation wage* w_R *depends positively both on non labor income V and on market attractiveness* θ.

In what follows, assuming a definite functional form for the individual’s utility function allows obtaining some interesting properties of the arts and non arts labor supply functions. In order to derive them we assume that the utility function is of the CES type.

Assumption (2). $U(c,l,h_A) = \left(x_1 c^\rho + x_2 l^\rho + x_3 {h_A}^\rho\right)^{1/\rho}$.

The CES utility function is more restrictive than a generic utility function but retains generality, as it includes as specific cases the linear, Cobb-Douglas and Leontieff types (when $\rho = 1$, $\rho \to 0$, and $\rho \to -\infty$ respectively). It is natural to assume that the elasticity of substitution is non-negative, and since it is equal to $1/(1-\rho)$, it must be that $\rho \le 1$. Symmetry is assumed for expositional purposes; i.e. different degrees of complementarities could be analyzed using a nested two-level CES function.

Equation **(8)** shows that for an interior solution we need $w - f_1 > 0$. This condition is weaker than Throsby's (1994) assumption that the wage in the labor market must be higher than the wage in the artistic market. It only implies that evaluated at the optimal artistic hours ${h_A}^*$, the hours-marginal arts income is lower than the labor market hourly wage. Using such utility function specification, it is straightforward to prove the following propositions (proofs are given in the appendix).

5.1 Part time artists' results

Proposition 3

Under A1 and A2, for a part time artist:

a. *Leisure is increasing in V.*
b. *Arts hours supply is increasing in V.*
c. *Non-arts market hours supply is decreasing in V.*

Proposition 4

Under A1 and A2, for a part time artist:

a. *An increase in the non-arts wage has an ambiguous effect on leisure.*
b. *An increase in the non-arts wage has an ambiguous effect on arts market hours supply.*
c. *An increase in the non-arts wage has an ambiguous effect on non-arts market hours supply.*

Proposition 5

Under A1 and A2, for a part time artist:

a. *An increase in market perceived quality θ has an ambiguous effect on leisure.*
b. *Arts market hours supply is increasing in market perceived quality θ.*
c. *Non-arts market hours supply is decreasing in market perceived quality θ.*

Assumption (3). $(f_{12} h_A)/f_2 \le 1-\rho$

Since f_2 is the market quality-marginal income from the artistic market, assumption 3 restricts the elasticity of market quality-marginal income with respect to hours to be below a certain threshold. It is not problematic if the utility function is Cobb-Douglas or Leontieff but for a linear utility function it would imply a negative elasticity.

Proposition 6.
Under A1 and A2, for a part time artist, A3 is a sufficient condition for leisure demand to be increasing in market perceived quality θ.

Income and substitution effects, part time artists

The effects of changes of non arts wages, non labor income and arts earning potential on arts and non arts hours can be interpreted as modified versions of the income and substitution effects in the traditional labor supply model (see proofs of propositions 3 to 6).

The derivatives of leisure and arts time with respect to non labor income are positive, and have a direct interpretation as a pure income effect (both are goods). On the contrary, the derivative of non arts time with respect to non labor income is negative, exactly as in the labor supply income effect result. The effect of non arts wage in arts time in leisure and arts time cannot be signed; however it can be decomposed as the sum of an always negative income effect and a substitution effect of ambiguous sign. This leads in turn to having an always negative income effect of non arts wage in non arts time but a substitution effect that cannot be signed.

5.2. Full time artists' results

Proposition 7.
Under A1 and A2, for a full time artist:

a. Arts hours supply is an increasing function of V
b. Arts hours supply is an increasing function of θ

Summarizing, we have obtained predictions as to the effect on time allocation of the exogenously given outside market opportunities, unearned income and arts earning potential. In what follows we try to test such implications using a small sample of artists.

6) Application

a) Data

Our data were obtained from a survey of a population of musicians members of a performing rights administration association. A sample of 474 artists was obtained from the records of the Uruguayan Society of Performing Artists (SUDEI) that collects on their behalf the performing rights from their recorded performances. Data were obtained directly from the artists in the sample, which were interviewed at their homes. The sample was stratified across several dimensions, including age, gender, artistic occupation and type of musical genre. Artistic occupation is defined in terms of largest time allocation. Musical genre was self defined. Table 2 shows basic descriptive statistics.

Table 2
Sample of performing musicians

Main artistic occupation	Cases	%
Composer	22	5.6
Arranger	12	3.1
Director	20	5.1
Producer	10	2.6
Player	190	48.5
Singer	87	22.2
Singer-songwriter	41	10.5
Improviser	1	0.3
Other	9	2.3
Total	392	100.0
Musical genre	**Cases**	**%**
Classical	53	13.5
Typical (tango)	33	8.4
Folklore	17	4.3
Popular	86	21.9
Tropical	51	13.0
Jazz	17	4.3
Brazilian	3	0.8
Murga	32	8.2
Carnival	11	2.8
Afro Uruguayan	22	5.6
Rock	31	7.9
Theater	3	0.8
Film or TV	6	1.5
Other	18	4.6
Not apply	9	2.3
None	392	100.0

Source: Survey of performing musicians, 2001

In table 3 we provide descriptive statistics for incomes, hours, labor and personal background variables for the musicians in our sample. Averages hide large heterogeneity between full time and part time artists, as well as between all year artistic workers and seasonal performers. We present separately data for full time artists and multiple job holders.

Table 3
Performing artists sample
Descriptive statistics (averages)

	Full time	Part time
Arts income	9176.8	3581.7
Non arts income		5889.4
Nonlabor income, arts assets (V_A)	787.8	783.2
Nonlabor income, non arts assets (V_n)	70.8	149.2
Arts hours	189.2	104.9
Non arts hours		141.8
Gender (% male)	85.4%	85.0%
Age	39.1	42.8
Art experience years	20.9	23.2
Arts formal education	64.9%	56.0%
Highest education attained secondary	48%	44%
Highest education attained technical	6%	12%
Highest education attained tertiary	41%	38%
Private sector worker in arts job	11%	15%
Public sector worker in arts job	24%	4%
Cooperative worker in arts job	8%	19%
Owners in arts job	5%	11%
Self employed in arts job	53%	50%

Source: Survey of performing musicians, 2001. Incomes are measured in October 2001 Uruguayan pesos; Hours and incomes are monthly averages.

Our artists' database shares the features that artists' data generally have. Censuses usually classify workers by occupation based on the largest contribution to labor income, and in many cases part time artists go undetected if there is not data on the second occupation. [9] In our case, since we have affiliates from a performing rights society, we have a bias towards those artists that do have an incentive to become members, i.e. those that have generated performing rights that arise from recorded performances. Another requisite to be admitted is proof of public performances. This biases our data towards more established artists.

Our recorded income values do not place the artists in our sample in the lowest quintiles of labor income distribution (as the "starving artist" view would suggest). Median artistic income (measured in monthly terms, correcting for seasonal activity) for full time artists is in our sample 6750 pesos, above median main occupation labor income for the October 2001 Uruguayan Household Survey, 6112 pesos. Artists in our sample seem to be more in the middle class than in the starving side of society.

b) Econometric strategy

To assess if our model is useful to describe artist's time allocation, we undertake the estimation of the parameters of the supply functions of hours in both the arts and non arts

[9] While in the Uruguayan Household survey sample only about 8% of workers hold a secondary job, in our artists' sample this fraction is about 50%.

markets, particularly the impacts of non earned income and non arts wages on hours dedicated to arts and non arts work. These will help to evaluate the effects of public subsidies to arts activity and provide valuable evidence regarding how arts work would react to changes in the economic environment and outside opportunities.

Our model yields two labor supply equations in arts and non arts activities, respectively:

$$h_A^p = h_A(w, V, \theta)$$

$$h_N^p = h_N(w, V, \theta)$$

We expect V to have a negative impact in non arts hours h_N and a positive effect on arts hours h_A, whereas perceived arts market quality θ should impact positively on arts time and negatively on non arts time. The net effect of substitution and income effects of non arts wage w changes in either supply equation remain to be empirically evaluated.

We will estimate the following system of equations:

$$h_A = \alpha_1 + \beta_1 w_i + \gamma_1 \theta_i + \delta_1 V_i + \varepsilon_{1i} \qquad \textbf{(12)}$$

$$h_N = \alpha_2 + \beta_2 w_i + \gamma_2 \theta_i + \delta_2 V_i + \varepsilon_{2i} \qquad \textbf{(13)}$$

Regarding θ_i, we postulate it is a function of a set of observable artistic individual characteristics plus an unobservable, random term that we may term “talent” or “box office appeal”.

$$\theta_i = aAK_i + bAXP_i + \sum_j d_j g_{ji} + \sum_k e_k Oc_{ki} + fGen_i + v_i$$

where AK_a is artistic human capital, measured by formal artistic education level attained, AXP_a is years of artistic work experience, g_j are dummy variables equal to one if the artistic activity belongs to each musical genre, Oc_k are dummy variables, equal to one for each of the artistic occupations and Gen_i is a gender dummy equal to one for males. Thus we do not include hourly arts earnings in our equation but rather estimate a reduced form based on earnings potential observed variables.

Substituting the following regression equations are obtained:

$$h_N = \alpha_2 + \beta_2 w_i + \gamma_2 \left[aAK_i + bAXP_i + \sum_j d_j g_{ji} + \sum_k e_k Oc_{ki} + fGen_i + v_i \right] + \delta_2 V_i + \varepsilon_{2i} \qquad \textbf{(14)}$$

$$h_A = \alpha_1 + \beta_1 w_i + \gamma_1 \left[aAK_i + bAXP_i + \sum_j d_j g_{ji} + \sum_k e_k Oc_{ki} + fGen_i + v_i \right] + \delta_1 V_i + \varepsilon_{1i} \qquad \textbf{(15)}$$

Each of these equations presents estimation problems. The dependent variable in the non arts hours equation is a response variable with a corner solution ($h_N = 0$) for fulltime artists that are near half of the sample. Non arts labour supply depends on non arts wage, but we likewise observe this variable only for part time artists.
Blundell, Ma Curdy and Meghir (2007) provide an up to date revision of the literature dedicated to the estimation of the single equation labour supply model and describe two approaches available for this problem problem. The first is the Heckman (1976, 1979) selectivity control estimation, and the second is using non parametric methods as surveyed by Robinson (1988). In this paper we use the Heckman methodology.

We only observe non arts wages for artists participating in both arts and non arts labour markets. This participation indicates that the non arts wage exceeds a reservation wage, defined as the shadow price of leisure time at zero non arts hours. Hence we can estimate a first stage non arts labor market participation model for part time artists, using a probit model in which participation is a function of all variables determining the difference between the observed wage and the reservation wage being greater than zero. The probability of having nonzero non arts hours is modeled as a function of the variables that determine if $w > w_R$, i.e. the determinants of non arts wages and the reservation wage.We can assume a Mincer-type equation for non arts wages like

$$ln\, w_i = \varphi_0 + \sum_j \varphi_j Educ_{ji} + \eta_1 EXP_i + controls \quad + \xi_i \qquad \textbf{(16)}$$

where *Educ* is education and *Exp* an experience measure. We can estimate the probability of being in the part time artists by a reduced form equation in which participation is a function of the set of variables that determine the non arts wage and the reservation wage. Our expression for the reservation wage tells us that participation depends on non wage income, as well as on the variables that determine θ_i are included as potentially affecting the probability of being a part time artist. It will also depend on variables that account for preferences, such as demographic indicators as gender, household head condition, if the artist has children, etc. Our estimated equation will be:

$$P(h_N > 0) = g\left(V_i, HK_{ni}, Age_i, Gen_i, HH_i, Ch_i, AK_{ai}, AXP_{ai}, g_{ji}, j = 1,..., J, Oc_{ki}, k = 1,...K\right) \qquad \textbf{(17)}$$

where V_i is unearned income, HK_n is non arts human capital, *Age* and Age^2 are age and the square of age, *Gen* is gender, *HH* is a dummy variable for household heads, *Ch* takes value 1 if the artist has children. Variables AK_a and AXP_a denote human capital and experience in arts, while *g* and *Oc* are musical genre and occupation and are all determinants of θ.

We can then obtain the inverse Mill's ratio and estimate sample selection corrected non arts hours equation. Using a probit functional form implies that the selection term is a nonlinear function of the selection equation right hand side variables, and hence identification of the second stage equation is guaranteed. In order not to rely only on normality assumptions implied by the probit, an exclusion restriction is also desirable for identification, i.e. some regressors in the non arts participation equation are not in the hours equations, which in our case holds. We then estimate the sample selection corrected equation **(14)** regressing the logs of arts and non arts hours in the same set of independent variables as in **(14)** and adding the selection bias term.

The arts hours function is different case, since arts hours are observed for all the sample. However, arts hours also depend on non arts wages, which are observed only for part time artists. Our estimation must take account of the missing wage problem. If we are willing to assume that wages are exogenous in the arts hour equation, following Blundell, MaCurdy and Meghir (2007) suggestion[10] that we may impute non arts wages for full time artists (for instance estimating a sample selection corrected Mincer-type equation as (16)) and estimate the model as if non arts wages were observed (correcting the standard errors for generated regressor bias[11]). However, endogeneity of wages in the arts hours equation can be corrected by using the same Heckman (1976, 1979) methodology: estimating consistently the parameters in equation **(15)** by running the sample selection corrected regression for part rime artists adding the selection bias term.

c) Results

To discuss our estimation of arts and non arts hours equations, we present first the results of the first stage probit model for non arts labor market participation in table 4.

Table 4
Probit estimates
Participation in non arts labor market

Number of obs	373	Pseudo R2		0,09
Wald chi2(5)	38,73	Log likelihood		-233,67
Prob > chi2	0,00			
	Coef	St. Error	z	P>|z|
Nonlabor income	-0,063	0,022	-2,86	0,004
Education	0,119	0,066	1,79	0,074
Age	0,041	0,016	2,6	0,009
Household head	0,377	0,191	1,97	0,048
Children	0,352	0,157	2,25	0,024
Arts education	-0,101	0,043	-2,36	0,018
Arts experience	-0,042	0,015	-2,87	0,004
Tropical music	0,583	0,208	2,8	0,005
Murga	0,647	0,281	2,31	0,021
Constant	-1,442	0,484	-2,98	0,003
Nonlabor income	-0,063	0,022	-2,86	0,004

[10] See Blundell, MaCurdy and Meghir (2007) p. 4680.
[11] See Woolridge (2002) section 6.1.

The estimation yields the expected signs of the non arts human capital and experience variables as well as those reflecting past choices or preferences such as household characteristics. Earnings power in arts as described by arts education and experience reduce the entry probability in the non arts labor market. Two musical genre dummies significantly affected entry in the non arts labor market, particularly being a murga musician, associated to Carnival and hence with a markedly seasonal artistic occupation.

Using the probit estimates we calculate the sample selection term to be included in the corrected estimates (following Heckman, 1979) arts hours. Results are displayed in table 5.

Table 5
Heckman selection model -- two-step estimates
(regression model with sample selection)
Non arts hours equation
Dependent variable is ln of arts hours

	Total	Censored	Uncensored	
Number of observations	370	195	175	
Wald chi2(4)	31.21			
Prob > chi2	0.00			
	Coef.	Std. Err	z	P>|z|
Log non labor income	0,008	0,023	0,370	0,709
Log non arts wage	-0,193	0,071	-2,720	0,007
Arts education	0,054	0,042	1,270	0,204
Arts experience	-0,001	0,006	-0,210	0,837
Gender	-0,053	0,208	-0,250	0,801
Constant	0,008	0,023	0,370	0,709
Mills' lambda	0,184	0,279	0,660	0,508

Table 5 shows that non arts wages have a statistically significant (and negative) influence in hours devoted to arts work. As to the economic significance of such impact, the implied elasticity of 0,19 reflects a modest decrease in working time outside art after a wage increase. Non labor income, though significantly affecting the participation decision, does not have a significant influence in the arts hours of artists. We do not find the Mill's ratio variable significant in this equation.

In table 6 we show the (sample selection corrected) estimates of parameters in non arts hours equation for part time artists.

Table 6
Heckman selection model -- two-step estimates
(regression model with sample selection)
Arts hours equation
Dependent variable is ln of non arts hours

Number of obs	370			
Censored obs	195			
Uncensored obs	175			
Wald chi2(4)	53.2			
Prob > chi2	0.00			
	Coef.	Std. Err	z	P>|z|
Log non labor income	0,009	0,018	0,490	0,625
Log non arts wage	-0,182	0,045	-4,050	0,000
Arts education	0,027	0,033	0,800	0,421
Arts experience	0,017	0,005	3,510	0,000
Gender	0,052	0,137	0,380	0,708
Constant	5,669	0,264	21,510	0,000
Mills' lambda	-0,805	0,214	-3,760	0,000

We do find a very similar in magnitude negative impact of the wage in non arts time, and in this case non labor income also does not exert a significant impact. We also find that variables affecting arts earning potential tend to perform generally poorly in both hours equations, though they affect significantly the participation decision. The Mill's ratio is significant in this estimation.

Our evidence seems to run counter Throsby's estimation of his "work preference" model. Throsby (1992) presents an estimation of labor supplies in both markets which yields a negative effect of the non arts wage on non arts hours and a positive effect in arts hours. However our results may not be directly comparable to his. In his paper, data are not wages but earnings that in turn are divided by hours to obtain "hourly earnings". Censoring of the observed dependent variables is not accounted for, and large mass points are observed at zero non art work hours for full time artists hence biasing estimated coefficients. Leisure is not included in the time constraint; hence arts time changes are transformed at a rate −1 in non arts time changes (though each time is measured as a proportion of total time). Rengers and Madden (2000) adapt Throsby's model and present an estimation in which hours and hourly earnings equations are estimated using Throsby's data. We believe that our specification allows for a more precise answer as to the effect of non arts wage in arts work time.

7) Conclusions

We have developed a model that allows analyzing relevant issues in artist's time allocation, particularly the responsiveness of their arts time to economic incentives. We applied such model to the analysis of a small sample of musicians to obtain empirical results to be contrasted with our model and with some results of previous literature.

Our model provides a utility maximization framework to analyze artist time allocation. It incorporates some realistic features such as the presence of arts time in the utility function and a nonlinear relation between arts earnings and arts hours, as well as specifically allowing for an explicit role of leisure in artist's decisions.

Differently from Throsby (1994), our model leaves undetermined a priori the effect of non arts wages in arts and non arts time allocation. This issue relates more to the relevance of the outside opportunities provided by non arts labor markets to artists than to arts public policy, since the latter rely basically in changing economic incentives to the dedication of artists by means of subsidies.

Our results show that non arts wages have a statistically significant negative effect in non arts time, of a modest economic size. However, they do affect negatively time allocation decisions to arts work. These results are not directly comparable with Throsby (1992) due to the methodological differences between them.

The evidence presented could be put in a perspective of simple backward bending labor supply curves in the non arts labor market, basically not different from those observed regularly for male workers (and our artists are largely male). Improvements of the state of the non arts labor market would induce them to work less outside art. This however would not translate in an increased dedication to arts work, since they also work less hours in art. Hence our results do not support some often cited intuition by which arts production displays a countercyclical behavior and critical times are accompanied by flourishing of artistic output and creation. If in times of crisis more artistic output is observed it may respond to exacerbated sensitivity but not to the change in the set of opportunities and then in time allocation.

No significant effect was found of non labor income in hours devoted to arts, though nonlabor income clearly affects (negatively) the non arts market participation decision. This can be potentially relevant to evaluate the role of subsidies in effectively changing dedication to arts activity.

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Appendix

Proof of Proposition 1

a. This follows from the strict inequality in equations (12) and (13) for the full time artist, which give $U_l/U_c > w$ and $\left(U_{h_A} + f_1 U_c\right)/U_c > w$. Only someone for which the constraint $h_A \geq 0$ is binding exactly at the margin, which for any continuous distribution of individuals across *w* has zero probability, would be affected.

b. Both w and θ affect (7) and (8). ■

Proof of Proposition 2

The reservation wage would be the wage w_R at which the individual chooses to work exactly *0* non arts hours. We add the condition $l = T - h_A$, and obtain $c = f\left(h_A, \theta\right) + V$. The reservation wage is obtained substituting in equation (7') from part time artist's first order conditions:

$$\frac{x_2\left(f(h_A,\theta)+V\right)^{1-\rho}}{x_1(T-h_A)^{1-\rho}}-w_R=0$$

It is immediate that w_R depends positively on market attractiveness θ and unearned income V.

■

For propositions 3 to 6, by assumption (2), $U(c,l,h_A)=\left(x_1c^{\rho}+x_2l^{\rho}+x_3{h_A}^{\rho}\right)^{\frac{1}{\rho}}$. Constrained utility maximization in the case of a part time artist, working with the first order conditions (equations (7'), (8') and (9'), gives the following system of equations:

$$\frac{x_2c^{1-\rho}}{x_1l^{1-\rho}}-w=0 \qquad \textbf{(7')}$$

$$\frac{x_3c^{1-\rho}}{x_1{h_A}^{1-\rho}}-(w-f_1)=0 \qquad \textbf{(8')}$$

$$c-wT+wl+wh_A-f(h_A,\theta)-V=0 \qquad \textbf{(9')}$$

Taking the total differential of equations (7'), (8') and (9') we get:

$$\frac{x_2(1-\rho)c^{-\rho}}{x_1l^{1-\rho}}dc-\frac{x_2(1-\rho)c^{1-\rho}}{x_1l^{2-\rho}}dl-dw=0 \qquad \textbf{(14)}$$

$$\frac{x_3(1-\rho)c^{-\rho}}{x_1{h_A}^{1-\rho}}dc+\left(f_{11}-\frac{x_3(1-\rho)c^{1-\rho}}{x_1{h_A}^{2-\rho}}\right)dh_A-dw+f_{12}d\theta=0 \qquad \textbf{(15)}$$

$$dc+wdl+(w-f_1)dh_A-(T-l-h_A)dw-f_2d\theta-dV=0 \qquad \textbf{(16)}$$

Writing in matrix form:

$$\begin{pmatrix} \frac{x_2(1-\rho)c^{-\rho}}{x_1l^{1-\rho}} & -\frac{x_2(1-\rho)c^{1-\rho}}{x_1l^{2-\rho}} & 0 \\ \frac{x_3(1-\rho)c^{-\rho}}{x_1{h_A}^{1-\rho}} & 0 & \left(f_{11}-\frac{x_3(1-\rho)c^{1-\rho}}{x_1{h_A}^{2-\upsilon}}\right) \\ 1 & w & w-f_1 \end{pmatrix}\begin{pmatrix} dc \\ dl \\ dh_A \end{pmatrix}=\begin{pmatrix} dw \\ dw+f_{12}d\theta \\ (T-l-h_A)dw+f_2d\theta+dV \end{pmatrix} \qquad \textbf{(17)}$$

To simplify notation we define

$$A = \begin{pmatrix} a & -b & 0 \\ c & 0 & -d \\ 1 & w & w - f_1 \end{pmatrix} = \begin{pmatrix} \frac{x_2(1-\rho)c^{-\rho}}{x_1 l^{1-\rho}} & -\frac{x_2(\rho-1)c^{1-\rho}}{x_1 l^{2-\rho}} & 0 \\ \frac{x_3(1-\rho)c^{-\rho}}{x_1 h_A^{\ 1-\rho}} & 0 & \left(f_{11} - \frac{x_3(\rho-1)c^{1-\rho}}{x_1 h_A^{\ 2-\upsilon}} \right) \\ 1 & w & w - f_1 \end{pmatrix} \qquad \textbf{(18)}$$

Where *a*, *b*, *c* and *d* are respectively

$$a = \frac{x_2(1-\rho)c^{-\rho}}{x_1 l^{1-\rho}} = \frac{(1-\rho)}{c}\frac{U_C}{U_l} > 0; \qquad b = \frac{x_2(\rho-1)c^{1-\rho}}{x_1 l^{2-\rho}} = \frac{(1-\rho)}{l}\frac{U_C}{U_l} > 0;$$

$$c = \frac{x_3(1-\rho)c^{-\rho}}{x_1 h_A^{\ 1-\rho}} = \frac{(1-\rho)}{c}\frac{U_C}{U_{h_A}} > 0; \qquad d = -\left(f_{11} - \frac{x_3(\rho-1)c^{1-\rho}}{x_1 h_A^{\ 2-\rho}} \right) = \frac{(1-\rho)}{h_A}\frac{U_C}{U_{h_A}} - f_{11} > 0$$

and rewrite the system of equations in (17) as

$$A \begin{pmatrix} dc \\ dl \\ dh_A \end{pmatrix} = \begin{pmatrix} dw \\ dw + f_{12} d\theta \\ (T - l - h_A)dw + f_2 d\theta + dV \end{pmatrix}$$

Lema 1. The determinant of matrix A is positive:

Proof: $|A| = bd + bc(w - f_1) + adw > 0$ ∎

Proof of Proposition 3

Setting $dw = d\theta = 0$ and dividing by dV, the system (5) becomes:

$$\begin{pmatrix} a & -b & 0 \\ c & 0 & -d \\ 1 & w & w - f_1 \end{pmatrix} \begin{pmatrix} dc/dV \\ dl/dV \\ dh_A/dV \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Applying Cramer's rule we get the following signs:

a. $\frac{dl}{dV} = \frac{1}{|A|} \begin{vmatrix} a & 0 & 0 \\ c & 0 & -d \\ 1 & 1 & w - f_1 \end{vmatrix} = \frac{1}{|A|} ad > 0$ b. $\frac{dh_A}{dV} = \frac{1}{|A|} \begin{vmatrix} a & -b & 0 \\ c & 0 & 0 \\ 1 & w & 1 \end{vmatrix} = \frac{1}{|A|} bc > 0$

The derivatives of both leisure and arts time with respect to non labor income are positive. Arts time and leisure are (given our utility function choice) normal goods; hence the income effect is in both cases positive.

b. $h_N = T - l - h_A$, therefore

$$dh_N/dV = -dl/dV - dh_A/dV = -(1/|A|)(ad+bc) < 0$$

This can be interpreted as a pure income effect in labor market hours. A richer individual would unequivocally work less in the labor market. It results directly from b. and c.

█

Proof of Proposition 4

Setting $dV = d\theta = 0$ and dividing by dw, the system (17) becomes:

$$\begin{pmatrix} a & -b & 0 \\ c & 0 & -d \\ 1 & w & w-f_1 \end{pmatrix} \begin{pmatrix} dc/dw \\ dl/dw \\ dh_A/dw \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ T-l-h_A \end{pmatrix}$$

Applying Cramer's rule we get the following signs:

a. $\dfrac{dl}{dw} = \dfrac{1}{|A|} \begin{vmatrix} a & 1 & 0 \\ c & 1 & -d \\ 1 & T-l-h_A & w-f_1 \end{vmatrix} = \dfrac{1}{|A|}\left[(a-c)(w-f_1) - d + ad(T-l-h_A)\right]$, cannot be signed.

In this case $\dfrac{dl}{dw} = \dfrac{1}{|A|}\left[(a-c)(w-f_1)-d\right] + \dfrac{dl}{dV}(T-l-h_A)$. The second term in the right hand side is again the income effect, which we know is positive. The first term in the right hand side is the substitution effect, reflecting the existence of two margins to optimize. The sign depends on the sign of the difference $(a-c)$ which depends on the difference in slopes of marginal rates of substitution between consumption and leisure and between consumption and arts time. If this difference is negative then the whole substitution effect is negative, and the net effect still depends on the size of both effects. If the difference $(a-c)$ is positive, then the substitution effect is positive and so it is the net effect.

b. $\dfrac{dh_A}{dw} = \dfrac{1}{|A|} \begin{vmatrix} a & -b & 1 \\ c & 0 & 1 \\ 1 & w & T-l-h_A \end{vmatrix} = \dfrac{1}{|A|}\left[(c-a)w - b + bc(T-l-h_A)\right]$, cannot be signed.

This case is symmetric to case a. We have

$$\frac{dh_A}{dw} = \frac{1}{|A|}\left[(c-a)w-b\right] + \frac{dh_A}{dV}(T-l-h_A),$$

in which the second term in the right hand side is the positive income effect. To have the first term on the right hand side positive, we need $(c-a)w-b>0$.

c. $h_N = T-l-h_A$, therefore

$$\frac{dh_N}{dw} = -\frac{dl}{dw} - \frac{dh_A}{dw} = -\frac{1}{|A|}\left[(a-c)f_1 + b + d - (ad+bc)(T - l - h_A)\right]$$, cannot be signed.

Here we have $\frac{dh_N}{dw} = \frac{1}{|A|}\left[(a-c)f_1 + b + d\right] - \left(\frac{dl}{dV} + \frac{dh_A}{dV}\right)(T - l - h_A)$

The income effect is always negative, and we cannot sign the substitution effect.

$\blacksquare$

Proof of Proposition 5

Setting $dV = dw = 0$ and dividing by $d\theta$, the system (17) becomes:

$$\begin{pmatrix} a & -b & 0 \\ c & 0 & -d \\ 1 & w & w - f_1 \end{pmatrix} \begin{pmatrix} dc/d\theta \\ dl/d\theta \\ dh_A/d\theta \end{pmatrix} = \begin{pmatrix} 0 \\ -f_{12} \\ f_2 \end{pmatrix}$$

Applying Cramer's rule we get the following signs:

a. $\frac{dl}{d\theta} = \frac{1}{|A|}\begin{vmatrix} a & 0 & 0 \\ c & -f_{12} & -d \\ 1 & f_2 & w - f_1 \end{vmatrix} = \frac{a}{|A|}\left[-f_{12}(w - f_1) + df_2\right]$ cannot be signed.

b. $\frac{dh_A}{d\theta} = \frac{1}{|A|}\begin{vmatrix} a & -b & 0 \\ c & 0 & -f_{12} \\ 1 & w & f_2 \end{vmatrix} = \frac{1}{|A|}\left[bf_{12} + bcf_2 + awf_{12}\right] > 0$

c. $h_W = T - l - h_A$, therefore

$$\frac{dh_W}{d\theta} = -\frac{dl}{d\theta} - \frac{dh_A}{d\theta} = -\frac{1}{|A|}\left[-af_{12}(w - f_1) + adf_2 + bf_{12} + bcf_2 + awf_{12}\right]$$
$$\frac{dh_W}{d\theta} = -\frac{1}{|A|}\left[af_{12}f_1 + adf_2 + bf_{12} + bcf_2\right] < 0$$

Proof of Proposition 6

a. $\frac{dl}{d\theta} = \frac{1}{|A|}\begin{vmatrix} a & 0 & 0 \\ c & -f_{12} & -d \\ 1 & f_2 & w - f_1 \end{vmatrix} = \frac{a}{|A|}\left[df_2 - f_{12}(w - f_1)\right]$

Substituting from equation (8') and the definition of *d* we obtain:

$$\frac{dl}{d\theta} = \frac{a}{|A|}\left[-\left(f_{11} - \frac{x_3(1-\rho)c^{1-\rho}}{x_1 h_A^{2-\upsilon}}\right)f_2 - f_{12}\frac{x_3 c^{1-\rho}}{x_1 h_A^{1-\rho}}\right] = \frac{a}{|A|}\left[-f_{11}f_2 + \frac{x_3 c^{1-\rho}}{x_1 h_A^{2-\rho}}\left(-f_{12}h_A + f_2(1-\rho)\right)\right]$$

Therefore, assumption 4 is a sufficient condition for $\frac{dl}{d\theta} > 0$.

■

Proof of proposition 7

For a full time artist, $T = l + h_A$. The individual's problem is now:

$$\underset{c,h_A}{Max} \quad U(c,h_A) = \left(x_1 c^\rho + x_2(T-h_A)^\rho + x_3 h_A{}^\rho\right)^{\frac{1}{\rho}}$$

subject to $c - f(h_a,\theta) - V = 0$

First order conditions are in this case:

$$\frac{x_3 c^{1-\rho}}{h_A{}^{1-\rho}} - \frac{x_2 c^{1-\rho}}{(T-h_A)^{1-\rho}} - f_1 = 0$$

$$c - f(h_a,\theta) - V = 0$$

Taking the total differential of both equations we get:

$$\left(\frac{x_3}{h_A{}^{1-\rho}} - \frac{x_2}{(T-h_A)^{1-\rho}}\right)(1-\rho)c^{-\rho}dc + \left\{(1-\rho)c^{1-\rho}\left(\frac{x_3 h_A{}^{-\rho}}{h_A{}^{2-2\rho}} + \frac{x_2(T-h_A)^{-\rho}}{(T-h_A)^{2-2\rho}}\right) - f_{11}\right\}dh_A - f_{12}d\theta = 0$$

$$dc - f_1 dh_A - f_2 d\theta - dV = 0$$

Writing in matrix form:

$$\begin{pmatrix} \left(\frac{x_3}{h_A{}^{1-\rho}} - \frac{x_2}{(T-h_A)^{1-\rho}}\right)(1-\rho)c^{-\rho} & (1-\rho)c^{1-\rho}\left(\frac{x_3}{h_A{}^{2-\rho}} + \frac{x_2}{(T-h_A)^{2-\rho}}\right) - f_{11} \\ 1 & -f_1 \end{pmatrix}\begin{pmatrix} dc \\ dh_A \end{pmatrix} = \begin{pmatrix} f_{12}d\theta \\ f_2 d\theta + dV \end{pmatrix}$$

To simplify notation we define

$$A = \begin{pmatrix} a & b \\ 1 & c \end{pmatrix} = \begin{pmatrix} \left(\frac{x_3}{h_A{}^{1-\rho}} - \frac{x_2}{(T-h_A)^{1-\rho}}\right)(1-\rho)c^{-\rho} & (1-\rho)c^{1-\rho}\left(\frac{x_3}{h_A{}^{2-\rho}} + \frac{x_2}{(T-h_A)^{2-\rho}}\right) - f_{11} \\ 1 & -f_1 \end{pmatrix}$$

Where *a*, *b* and *c* are respectively

$$a=\left(\frac{x_3}{h_A^{\,1-\rho}}-\frac{x_2}{(T-h_A)^{1-\rho}}\right)(1-\rho)c^{-\rho}; \qquad b=(1-\rho)c^{1-\rho}\left(\frac{x_3}{h_A^{\,2-\rho}}+\frac{x_2}{(T-h_A)^{2-\rho}}\right)-f_{11};$$

$$c=-f_1<0$$

The sign of a is the sign of $\left(\frac{x_3}{h_A^{\,1-\rho}}-\frac{x_2}{(T-h_A)^{1-\rho}}\right)$; from first order conditions we obtain that $U_{h_A}=U_C f_1>0$. As $U_{h_A}=U^{1-\rho}\left(\frac{x_3}{h_A^{\,1-\rho}}-\frac{x_2}{(T-h_A)^{1-\rho}}\right)$, it must be $\left(\frac{x_3}{h_A^{\,1-\rho}}-\frac{x_2}{(T-h_A)^{1-\rho}}\right)>0$. Hence $a>0$.

As the sign of $-f_{11}$ is unequivocally positive, the sign of b depends on the sign of $\left(x_3/h_A^{\,2-\rho}+x_2/(T-h_A)^{2-\rho}\right)$, hence $b>0$.

The system of equations in (17) can be rewritten as

$$A\begin{pmatrix} dc \\ dh_A \end{pmatrix}=\begin{pmatrix} f_{12}d\theta \\ f_2 d\theta+dV \end{pmatrix}$$

The determinant of the matrix A is given by $|A|=ac-b$; then $|A|<0$. Setting $d\theta=0$ and dividing by dV, the system (5) becomes:

$$\begin{pmatrix} a & b \\ 1 & c \end{pmatrix}\begin{pmatrix} dc/dV \\ dh_A/dV \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Applying Cramer's rule we get $\frac{dh_A}{dV}=\frac{1}{|A|}\begin{vmatrix} a & 0 \\ 1 & 1 \end{vmatrix}=\frac{a}{|A|} \quad >0$

Setting $dV=0$ and dividing by $d\theta$, the system (5) becomes:

$$\begin{pmatrix} a & b \\ 1 & c \end{pmatrix}\begin{pmatrix} dc/d\theta \\ dh_A/d\theta \end{pmatrix}=\begin{pmatrix} f_{12} \\ f_2 \end{pmatrix}$$

Applying Cramer's rule we get $\frac{dh_A}{d\theta}=\frac{1}{|A|}\begin{vmatrix} a & f_{12} \\ 1 & f_2 \end{vmatrix}=\frac{1}{|A|}(af_{12}-f_2) \quad >0$.

$\blacksquare$